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*PR* perpendicular to the opposite side of the angle. Mark off a distance equal to $2\overline{OP}$ on a straight-edge passing through *O*, and adjust the straight-edge so that one mark falls on line *PQ* at *M*, and the other on line *PR* at *N*. Draw the line *NOM*, which will trisect the angle.

*Proof:* Complete the rectangle *PMKN*, and draw the other diagonal *PK*. Then $\overline{OP}=\frac{1}{2}\overline{MN}=\frac{1}{2}\overline{PK}=\overline{PC}=\overline{CM}$.

Hence, $\theta=\alpha+\phi=\beta+\phi=\gamma+\delta+\phi=2\delta+\phi=2\phi+\phi=3\phi$.

# DEPARTMENTS.

## SOLUTIONS OF PROBLEMS.

### ALGEBRA.

283. Proposed by **G. B. M. ZERR, A. M., Ph. D.,** Parsons, W. Va.

Solve $w+x+y+z=4a$, $w^2+x^2+y^2+z^2=4a^2+4b^2$, $w^3+x^3+y^3+z^3=4a^3+12ab^2$, $w^4+x^4+y^4+z^4=4a^4+4b^4+4c^4+24a^2b^2$.

Solution by **DR. L. E. DICKSON,** Associate Professor of Mathematics, The University of Chicago.

The following method applies equally well to the corresponding equations with arbitrary constant terms. We are given $s_1$, $s_2$, $s_3$, $s_4$, where $s_n$ is the sum of the *n*th powers of *w*, *x*, *y*, *z*. Hence the latter are, by Newton's identities, the roots of the following quartic:

$$\xi^4-4a\xi^3+(6a^2-2b^2)\xi^2+(4ab^2-4a^3)\xi+a^4+b^4-2a^2b^2-c^4=0.$$

To obtain the reduced quartic, set $\xi=\eta+a$. Then

$$\eta^4-2b^2\eta^2+b^4-c^4=0, \quad (\eta^2-b^2)^2=c^4.$$

Hence, the 24 sets of solutions are given by the arrangements of $a\pm\sqrt{(b^2\pm c^2)}$.

Similarly solved by G. B. M. Zerr and J. Scheffer.

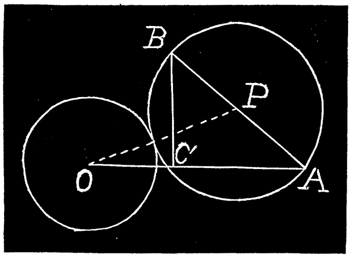
### GEOMETRY.

312. Proposed by **F. H. SAFFORD, Ph. D.,** The University of Pennsylvania, Philadelphia, Pa.

A variable circle passes through a fixed point and is tangent to a given circle. If a diameter of the first circle passes through the fixed point, find the locus of its other extremity.

**Solution by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.**

Let $O$ be the center of the given circle, radius$=r$, $A$ the given point, $OA=b$, $OC=x$, $BC=y$. Denoting the radius of the variable circle $P$ by $\rho$, then



$$(r+\rho)^2=\tfrac{1}{2}y^2+\tfrac{1}{4}(b+x)^2 \text{ and } 4\rho^2=y^2+(b-x)^2.$$

Eliminating $\rho$, we obtain the equation of the locus of $B$,

$$y^2=\frac{b^2-r^2}{r^2}(x^2-r^2),$$

a hyperbola, with semi-conjugate axes $r$ and $\sqrt{(b^2-r^2)}$. Consequently, $A$ will be a focus.

Also solved by J. S. Brown.

313. **Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.**

Prove that an algebraic curve of odd degree which is symmetrical with respect to a center has the center on the curve.

I. **Solution by DR. L. E. DICKSON, Associate Professor of Mathematics, The University of Chicago.**

By a transformation of coordinates we may assume that the center is at the origin. If the resulting equation of odd degree is $f(x, y)=0$, then must $f(-x, -y)=0$ for all values $x$, $y$ satisfying $f(x, y)=0$. But the terms of highest degree in $f(x, y)$ change sign when $x$ and $y$ are changed in sign. Hence $f(-x, -y)\equiv -f(x, y)$, so that the terms of even degree vanish. In particular, the constant term is zero, so that the origin lies on the curve.

II. **Solution by G. W. GREENWOOD, M. A., Dunbar, Pa.**

If in the equation of any locus we put $-x$, $-y$ for $x$, $y$, respectively, we get a locus symmetrical to the given locus with respect to the origin. In case the locus is symmetrical to itself, the latter equation will represent the same locus and differ from the original equation only by a constant multiplier. Let

$$0=u_0+u_1+u_2+u_3+\ldots+u_r+\ldots$$

be the equation of a locus symmetrical with respect to the origin, $u_n$ denoting the terms of degree $n$, and $r$ being an *odd* integer. Another equation to this locus, obtained by putting $-x$, $-y$ for $x$, $y$, respectively, and changing the sign of each term, is

$$0=-u_0+u_1-u_2+u_3-\ldots+u_r-\ldots$$

Since each equation contains the term $u_r$, the constant multiplier is unity.